1. Half the surface of a transparent sphere of refractive index 2 is silvered. A narrow, parallel beam of light is incident on
the unsilvered surface, symmetrically with respect to the silvered part. The light finally emerging from the sphere will
be a :

(A*) Parallel beam  (B) Converging beam  (C) Slightly divergent beam  (D) Widely divergent beam

Sol.
Refraction from surface 1 : \[ \frac{2}{v_1} - \frac{1}{\infty} = \frac{(2-1)}{(+R)} \]
\[ \Rightarrow v_1 = +2R \]
Reflection from silvered surface 2 : Since object is at pole so image also is at pole.
Refraction from surface 2 : \[ \frac{1}{v_2} - \frac{2}{-2R} = \frac{1-2}{(-R)} \]
\[ \Rightarrow v_2 = \infty \]
Therefore the final emergent light will emergent as parallel beam.

2. A transparent sphere of radius \( R \) and refractive index \( \mu \) is kept in air. At what distance from the surface of the sphere
should a point object be placed so as to form a real image at the same distance from the sphere ?

(A) \( \frac{R}{\mu} \)  (B) \( \mu R \)  (C*) \( \frac{R}{\mu-1} \)  (D) \( \frac{R}{\mu+1} \)

Sol.
This question can be solved by symmetry as is shown in figure. If \( d \) be the distance of object from first surface then
refracted ray from first surface should become parallel.
\[ \frac{\mu - 1}{\infty} \frac{(-d)}{(+R)} = \frac{(\mu-1)}{R} \]
\[ \Rightarrow 0 + \frac{1}{d} = \frac{(\mu-1)}{R} \]
\[ \Rightarrow d = \frac{R}{(\mu-1)} \text{ Ans.} \]

3. A ray of monochromatic light is incident on the plane surface of separation between two media \( x \) & \( y \) with angle of
incidence \( i \) in the medium \( x \) and angle of refraction \( r \) in the medium \( y \). The graph shows the relation between \( \sin r \)
and \( \sin i \):

(A) The speed of light in the medium \( y \) is \((3)^{1/2}\) times then in medium \( x \).  
(B*) The speed of light in the medium \( y \) is \((1/3)^{1/2}\) times then in medium \( x \).  
(C) The total internal reflection can take place when the incidence is in \( x \)  
(D*) The total internal reflection can take place when the incidence is in \( y \).

Sol.
From graph
\[ \frac{\sin r}{\sin i} = \tan 30^\circ = \frac{1}{\sqrt{3}} \]
\[ \Rightarrow \sin i = \sqrt{3} \sin r \]
From snell’s law :
\[ \mu_x \sin i = \mu_y \sin r \]
\[ \sqrt{3} \sin r = \frac{c}{v_y} \sin r \]

\[ v_y \sqrt{3} = v_x \]

4. A point source is placed at a depth \( h \) below the surface of water (refractive index = \( \mu \)). The medium above the surface of water is air (\( \mu = 1 \)). Find the area on the surface of water through which light comes in air from water.

**Ans.** \[ \frac{\pi h^2}{\mu^2 - 1} \]

**Sol.**

\[ u \times \sin \theta = 1 \times \sin 90^\circ \]

\[ \sin \theta = \frac{1}{\mu} \]

\[ \Rightarrow \tan \theta = \frac{1}{\sqrt{\mu^2 - 1}} \]

\[ R = h \tan \theta = \frac{1}{\sqrt{\mu^2 - 1}} \]

\[ \therefore \text{Area} = \pi R^2 = \frac{\pi h^2}{(\mu^2 - 1)} \]

\[ \therefore \text{Area} = \frac{\pi h^2}{(\mu^2 - 1)} \quad \text{Ans.} \]

\[ \therefore v_y = \frac{1}{\sqrt{3}} v_x \]

5. Given that, \( v \) velocity of light in quartz = \( 1.5 \times 10^8 \) m/s and velocity of light in glycerine = \( (9/4) \times 10^8 \) m/s. Now a slab made of quartz is placed in glycerine as shown. The shift of the object produced by slab is :

(A*) 6 cm
(B) 3.55 cm
(C) 9 cm
(D) 2 cm

**Sol.**

Normal shift = \( h_{act} \left( 1 - \frac{\mu_x}{\mu_d} \right) \)

here:

\[ h_{act} = 18 \text{ cm} \]

\[ h_t = h_{glycerine} = \frac{3 \times 10^8}{(9/4) \times 10^8} = \frac{4}{3} \]

\[ h_d = h_{quartz} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2 \]

\[ \therefore \text{Normal shift} = 18 \left( 1 - \frac{4/3}{2} \right) = 18 \left( 1 - \frac{4}{6} \right) \]

\[ = 18 \left( \frac{1}{3} \right) = 6 \text{ cm} \]

\[ \therefore \text{Normal shift} = 6 \text{ cm} \quad \text{Ans.} \]
6. Two refracting media are separated by a spherical interface as shown in the figure. $P P'$ is the principal axis, $\mu_1$ and $\mu_2$ are the refractive indices of medium of incidence and medium of refraction respectively. Then:

(A) If $\mu_2 > \mu_1$, then there cannot be a real image of real object
(B) If $\mu_2 > \mu_1$, then there cannot be a real image of virtual object
(C) If $\mu_1 > \mu_2$, then there cannot be a virtual image of virtual object
(D) If $\mu_1 > \mu_2$, then there cannot be a real image of real object

Sol. Object is in medium $\mu_1$ at a distance $x$ from pole

$$\frac{\mu_2}{v} \left(-x\right) = \frac{\mu_2 - \mu_1}{-R}$$

$$\Rightarrow \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R} \frac{\mu_1}{x}$$

$$\Rightarrow v = \frac{\mu_2 R x}{(\mu_1 - \mu_2) x - (\mu_1) R} = \frac{\mu_2 R}{\mu_1 \left[\frac{1 - \mu_2}{\mu_1}^{-R} \frac{x}{x}\right]}$$

**Case-1:** Object is real $\Rightarrow x$ negative $\Rightarrow$ Let $x = -d$

So (1A) Image is real $\Rightarrow v$ positive $\Rightarrow \left(1 - \frac{\mu_2}{\mu_1}\right) \frac{R}{d} > 0$

$$\Rightarrow 1 - \frac{\mu_2}{\mu_1} > \frac{R}{d}$$

$$d > \frac{\mu_1 R}{(\mu_1 - \mu_2)}$$

(1B) Image is virtual $\Rightarrow v$ negative $\Rightarrow \left(1 - \frac{\mu_2}{\mu_1}\right) \frac{R}{d} < 0$

$$\Rightarrow 1 - \frac{\mu_2}{\mu_1} < \frac{R}{d}$$

**Case-2:** Object is virtual $\Rightarrow x$ negative $\Rightarrow$ Let $x = -d$

So (2A) Image is real $\Rightarrow v$ positive $\Rightarrow \left(1 - \frac{\mu_2}{\mu_1}\right) \frac{R}{d} > 0$

$$\Rightarrow \left(1 - \frac{\mu_2}{\mu_1}\right) \frac{R}{d} < 0$$

$$d < \frac{\mu_1 R}{(\mu_2 - \mu_1)}$$

(2B) Image is virtual $\Rightarrow v$ negative $\Rightarrow \left(1 - \frac{\mu_2}{\mu_1}\right) \frac{R}{d} < 0$

$$\Rightarrow \left(1 - \frac{\mu_2}{\mu_1}\right) \frac{R}{d} > 0$$

$$d > \frac{\mu_1 R}{(\mu_2 - \mu_1)}$$

Looking at four cases

If: $\mu_2 \geq \mu_1 \Rightarrow$ Real object can not have real image

If: $\mu_2 \leq \mu_1 \Rightarrow$ Virtual object can not have virtual image

7. A spherical surface of radius 30 cm separates two transparent media $A$ and $B$ with refractive Indices 4/3 and 3/2 respectively. The medium $A$ is on the convex side of the surface. Where should a point object be placed in medium $A$ so that the paraxial rays become parallel after refraction at the surface?

**Ans.** [240 cm away from the separating surface]
8. A ray incident at a point at an angle of incidence of $60^\circ$ enters a glass sphere of $\mu = \sqrt{3}$ and it is reflected and refracted at the farther surface of the sphere. The angle between reflected and refracted rays at this surface is:

(A) $50^\circ$  
(B*) $90^\circ$  
(C) $60^\circ$  
(D) $40^\circ$

**Sol.**

$$\sin 60^\circ = \sqrt{3} \sin r$$

$$\sin r = \frac{1}{2}$$

$$r = 30^\circ$$

$$\theta = 180 - (60 + r) = 90^\circ$$

9. A ray of light is incident on a parallel slab of thickness $t$ and refractive index $n$. If the angle of incidence $\theta$ is small, then the displacement in the incident and emergent ray will be:

(A*) \( \frac{\theta(n-1)}{n} \)  
(B) \( \frac{\theta}{n} \)  
(C) \( \frac{\theta n}{n-1} \)  
(D) None

**Sol.**

$$\sin \theta = n \sin r$$

$$x = t \sin r$$

$$x + y = t \sin \theta$$

$$\Rightarrow \quad \frac{\sin \theta}{n} + y = t \sin \theta$$

and

$$\frac{d}{y} = \cos \theta$$

$$\sin \theta = 0$$

$$\cos \theta = 1$$

$$\Rightarrow \quad \frac{\theta}{n} + d = \theta$$

or

$$d = \frac{\theta(1 - \frac{1}{n})}{n}$$

$$= \frac{\theta(n-1)}{n}$$

10. A stationary swimmer $S$, inside a liquid of refractive index $\mu_1$, is at a distance $d$ from a fixed point $P$ inside the liquid. A rectangular block of width $t$ and refractive index $\mu_2$ ($\mu_2 < \mu_1$) is now placed between $S$ and $P$. $S$ will observe $P$ to be at a distance:

(A) $d - t \left( \frac{\mu_1 - 1}{\mu_2 - 1} \right)$  
(B) $d - t \left( 1 - \frac{\mu_2}{\mu_1} \right)$  
(C) $d + t \left( 1 - \frac{\mu_2}{\mu_1} \right)$  
(D*) $d + t \left( \frac{\mu_1 - 1}{\mu_2 - 1} \right)$

**Sol.**

$$\text{distance} = (d - t) + t \left( \frac{\mu_2}{\mu_1} \right)$$

$$= d + t \left( \frac{\mu_1}{\mu_2} - 1 \right)$$
1. A ray of light falls on a transparent sphere with centre at $C$ as shown in figure. The ray emerges from the sphere parallel to line $AB$. The refractive index of the sphere is:

(A) $\sqrt{2}$
(B) $\sqrt{3}$
(C) $\frac{3}{2}$
(D) $\frac{1}{2}$

**Sol.**

\[
i_2 = r_1 \quad \text{(isosceles triangle CDE)}
\]

\[
\Rightarrow \quad r_2 = 60^\circ
\]

\[
\Rightarrow \quad r_1 + x_2 = 60^\circ
\]

or

\[
\Rightarrow \quad r_1 = 30^\circ
\]

\[
\Rightarrow \quad \mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}
\]

2. A beam of diameter $'d'$ is incident on a glass hemisphere as shown. If the radius of curvature of the hemisphere is very large in comparison to $'d'$, then the diameter of the beam at the base of the hemisphere will be:

(A) $\frac{3}{4}d$
(B) $d$
(C) $\frac{d}{3}$
(D) $\frac{2}{3}d$

**Sol.**

For objects at infinity image will be formed at:

\[
\frac{3/2}{V} = \frac{1/2}{R}
\]

or

\[
V = 3R.
\]

from similar triangles $ABC$ and $ADE$

\[
\frac{D'}{2R} = \frac{D}{3R}
\]

\[
\Rightarrow \quad D' = \frac{2}{3}D.
\]

3. Rays incident on an interface would converge 10 cm below the interface if they continued to move in straight lines without bending. But due to refraction, the rays will bend and meet somewhere else. Find the distance of meeting point of refracted rays below the interface, assuming the rays to be making small angles with the normal to the interface.

**Ans.** [25 cm]
Sol.

\[
\sin i = \frac{x}{\sqrt{10^2 + x^2}}
\]
\[
\sin r = \frac{x}{\sqrt{y^2 + x^2}}
\]
\[
\frac{\sin i}{\sin r} = \frac{5}{2} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + 10^2}}
\]

For small \( i \) and \( r \)

\[x \ll 10\] and \( x \ll y\)

\[\Rightarrow \quad \frac{5}{2} = \frac{y}{10}\]

or \( y = 25 \text{ cm} \)

4. Find the apparent depth of the object seen by observer \( A \) ?

Ans. \( \frac{68}{3} \text{ cm} \)

Sol. Apparent depth

\[
\text{Apparent depth} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}
\]

\[
= \frac{25}{1.5} + \frac{15}{(2.5)/(1.5)}
\]

\[
= \frac{50}{3} + 6 = \frac{68}{3}
\]

5. A point source of light at the surface of a sphere causes a parallel beam of light to emerge from the opposite surface of the sphere. The refractive index of the material of the sphere is :

(A) 1.5

(B) 5/3

(C*) 2

(D) 2.5

Sol.

\[
\mu \sin \theta = 1 \times \sin 20
\]

\[
\mu \sin \theta = 2 \sin \theta \cos \theta
\]

\[
\mu = 2 \cos \theta
\]

for \( \theta \) is small

\[
\mu = 2
\]

6. An object is placed 30 cm (from the reflecting surface) in front of a block of glass 10 cm thick having its farther side silvered. The final image is formed at 23.2 cm behind the silvered face. The refractive index of glass is :

(A) 1.41

(B) 1.46

(C*) 100/132

(D) 1.61

Sol.

\[
BI_1 = (10 + 20\mu)
\]

\[
BI_2 = (10 + 20\mu)
\]

\[
AI_2 = (20 + 20\mu)
\]

\[
AI_3 = (23.2) + 10 = 33.2 = (20 \times 20\mu) \frac{1}{\mu}
\]

\[
\mu = \frac{200}{132}
\]

7. The \( x-y \) plane is the boundary between two transparent media. Media-1 with \( z > 0 \) has refractive index \( \sqrt{2} \) and medium – 2 with \( z < 0 \) has a refractive index \( \sqrt{3} \). A ray of light in medium-1 given by the vector \( \vec{A} = 6\sqrt{3}i + 8\sqrt{3}j - 10k \) is incident on the plane of separation. Find the unit vector in the direction of refracted ray in medium –2.
Ans. \[ \mathbf{r} = \frac{3}{5\sqrt{2}} \mathbf{i} + \frac{2\sqrt{2}}{5} \mathbf{j} - \frac{1}{\sqrt{2}} \mathbf{k} \] (angle of incidence = 60º, \( r = 45º \))

Sol. Unit vector of in \( xy \) plane for incident ray and unit vector in \( xy \) plane for refracted will be same plane because they need to be in same plane.

Here component of incident ray in \( xy \) plane is \( 6\sqrt{3} \mathbf{i} + 5\sqrt{3} \mathbf{j} \)

Unit vector of incident ray in \( xy \) plane is \( \frac{3}{5} \mathbf{i} + \frac{4}{5} \mathbf{j} \) angle with \( x \) axis is \( \alpha = 53º \)

Component of unit refracted vector in \( xy \) plane is \( \frac{1}{\sqrt{2}} \)

Here refracted ray is \( \frac{1}{\sqrt{2}} \cos \theta \mathbf{i} + \frac{1}{\sqrt{2}} \sin \theta \mathbf{j} - \frac{1}{\sqrt{2}} \mathbf{k} \)

\[ \frac{3}{5\sqrt{2}} \mathbf{i} + \frac{2\sqrt{2}}{5} \mathbf{j} - \frac{1}{\sqrt{2}} \mathbf{k} \]

8. A ray of light passes through four transparent media with refractive indices \( \mu_1, \mu_2, \mu_3, \) and \( \mu_4 \) as shown in the figure. The surfaces of all media are parallel. If the emergent ray \( CD \) is parallel to the incident ray \( AB \), we must have:

(A) \( \mu_1 = \mu_2 \)
(B) \( \mu_2 = \mu_3 \)
(C) \( \mu_3 = \mu_4 \)
(D*) \( \mu_4 = \mu_1 \)

Sol. If \( CD \) is parallel to \( AB \) then \( \mu_1 = \mu_4 \)

9. An observer can see through a pin hole the top of a thin rod of height \( h \), placed as shown in the figure. The beaker height is \( 3h \) and its radius is \( h \). When the beaker is filled with a liquid up to a height \( 2h \), he can see the lower end of the rod. Then the refractive index of the liquid is:

(A) \( \frac{5}{2} \)
(B*) \( \frac{\sqrt{5}}{2} \)
(C) \( \frac{3}{2} \)
(D) \( \frac{3}{2} \)

Sol. \( \tan \alpha = \frac{h}{h} = 1 \Rightarrow \alpha = 45º \)

\[ \tan \theta = \frac{h}{2h} = \frac{1}{2} \Rightarrow \sin \theta = \frac{1}{\sqrt{5}} \]

\( \sin \alpha = \frac{1}{\sqrt{2}} \)

Snell's law \( \mu \times \sin \theta = 1 \times \sin \alpha \)

\[ \mu = \frac{1}{\sqrt{5}} \]

Ans. \( \mu = \frac{\sqrt{5}}{2} \)

10. A solid, transparent sphere has a small, opaque dot at its centre. When observed from outside, the apparent position of the dot will be:

(A) Closer to the eye than its actual position
(B) Farther away from the eye than its actual position
(C*) The same as its actual position
(D*) Independent of the refractive index of the sphere

Sol. \( u = -R \)

\( v = ? \)
\[
\frac{1}{v} \frac{\mu}{(-R)} = \frac{1 - \mu}{(-R)}
\]

\[
\frac{1}{v} + \frac{\mu}{R} = \frac{1 - \mu}{R}
\]

\[v = -R\]

Hence dot will appear at its original position.
1. Given that, velocity of light in quartz = $1.5 \times 10^8$ m/s and velocity of light in glycerine = $(9/4) \times 10^8$ m/s. Now a slab made of quartz is placed in glycerine as shown. What is the shift produced by slab?

(A*) 6 cm  
(B) 3.55 cm  
(C) 9 cm  
(D) 2 cm

**Sol.**

$$\text{Shift} = t \left(1 - \frac{\mu_{\text{rare}}}{\mu_{\text{denser}}}\right)$$

$$\mu_{\text{rare}} = \mu_{\text{glycerine}} = \frac{3 \times 10^8}{9/4 \times 10^8} = \frac{4}{3}$$

$$\mu_{\text{denser}} = \mu_{\text{quarter}} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2$$

$$\text{shift} = 18 \left(1 - \frac{4/3}{2}\right) = 6 \text{ cm}$$

2. A rectangular glass slab $ABCD$, of refractive index $n_1$, is immersed in water of refractive index $n_2$ ($n_1 > n_2$). A ray of light in incident at the surface $AB$ of the slab as shown. The maximum value of the angle of incidence $\alpha_{\text{max}}$ such that the ray comes out only from the other surface $CD$ is given by:

(A*) $\sin^{-1} \left[ \frac{n_1 \cos \left( \sin^{-1} \frac{n_2}{n_1} \right)}{n_2} \right]$  
(B) $\sin^{-1} \left[ \frac{n_1 \cos \left( \sin^{-1} \frac{1}{n_2} \right)}{n_2} \right]$  
(C) $\sin^{-1} \left[ \frac{n_1}{n_2} \right]$  
(D) $\sin^{-1} \left[ \frac{n_2}{n_1} \right]$

**Sol.**  
Condition will be satisfied it on lateral surface $T.I.R$ takes place. Forthat $90 - \theta \geq \theta_c$  
$$\Rightarrow \quad \theta \leq 90 - \theta_c$$

$$\Rightarrow \quad \sin \theta \leq \sin(90 - \theta_c) \quad [\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)]$$

$$\sin \leq \cot (\theta_c) \Rightarrow [\sin \theta \leq \cos \sin^{-1} \left( \frac{n_2}{n_1} \right)]$$

Snell's law
\[ n_2 \sin \theta = n_1 \sin \theta \Rightarrow \frac{n_2}{n_1} \sin \theta = \sin \theta \leq \cos \left[ \sin^{-1} \left( \frac{n_1}{n_2} \right) \right] \]

\[ \Rightarrow \sin \theta \leq \frac{n_1}{n_2} \cos \left[ \sin^{-1} \left( \frac{n_1}{n_2} \right) \right] \]

\[ \therefore \quad \alpha_{\text{max}} = \sin^{-1} \left[ \frac{n_1}{n_2} \cos \left[ \sin^{-1} \left( \frac{n_1}{n_2} \right) \right] \right] \]

3. A light beam is traveling from Region I to Region IV (Refer figure). The refractive index in Regions I, II, III and IV are \( n_0 \), \( n_0/2 \), \( n_0/6 \) and \( n_0/8 \), respectively. The angle of incidence \( \theta \) for which the beam just misses entering Region IV is Figure :

(A) \( \sin^{-1} \left( \frac{3}{4} \right) \)  
(B) \( \sin^{-1} \left( \frac{1}{8} \right) \)  
(C) \( \sin^{-1} \left( \frac{1}{4} \right) \)  
(D) \( \sin^{-1} \left( \frac{1}{3} \right) \)

Sol. \[
\begin{align*}
n_0 \sin \theta &= \frac{n_0}{2} \sin \theta_1 = \frac{n_0}{6} \sin \theta_2 \Rightarrow \sin \theta = \frac{1}{6} \sin \theta_2, \\
\theta_2 \geq (\theta_2)_{3-4} &\Rightarrow \sin \theta_2 \geq \frac{n_0/8}{n_0/6} \Rightarrow \sin \theta \geq \frac{3}{4} \\
\therefore \quad \sin \theta \geq \frac{1}{6} \times \frac{3}{4} &\Rightarrow \sin \theta \geq \frac{1}{8} \\
\therefore \quad \theta_{\text{min}} = \sin^{-1} \left( \frac{1}{8} \right) \end{align*}
\]

4. A hemispherical portion of the surface of a solid glass sphere (\( \mu = 1.5 \)) of radius \( r \) is silvered to make the inner side reflecting. An object is placed on the axis of the hemisphere at a distance \( 3r \) from the centre of the sphere. The light from the object is refracted at the unsilvered part, then reflected from the silvered part and again refracted at the unsilvered part. Locate the final image formed.

Ans. [At the pole of reflecting surface of the sphere]

Sol. Refraction at single surface no. (1)

\[
\frac{3}{2} - \frac{1}{(-2r)} = \frac{3}{2} - \frac{1}{+r} \\
\Rightarrow \quad v_1 \rightarrow \infty
\]

Reflection from silvered concave surface (2)

\[
\frac{1}{v_2} + \frac{1}{\infty} = -\frac{2}{r} \Rightarrow v_2 = -\frac{r}{2}
\]

Refraction from surface (1) emerging out:

\[
\frac{1}{v_3} - \frac{3/2}{(-3r/2)} = \frac{(1 - 3/2)}{(-r)} \Rightarrow v_3 = -2r
\]

This means that final image is formed at pole of reflecting surface (2)
5. In ray of light \((GH)\) is incident on the glass-water interface \(DC\) at an angle \(i\). It emerges in air along the water-air interface \(EF\) (see figure). If the refractive index of water \(\mu_w\) is \(4/3\), the refractive index of glass \(\mu_g\) is:

(A) \(\frac{3}{4\sin i}\) 
(B) \(\frac{1}{\sin i}\) 
(C) \(\frac{3\sin i}{4}\) 
(D) \(\frac{4}{3\sin i}\)

Sol. Refraction at \(CD\):
\[
\mu_g \times \sin i = \mu_w \times \sin r
\]
Refraction at \(F_2F\):
\[
\mu_w \times \sin r = 1 \times \sin 90^\circ
\]
\[\therefore \quad \mu_g \sin i = 1 \times \sin 90^\circ\]
\[\mu_g = \frac{1}{\sin i}\]

6. A parallel beam of light is incident normally on the flat surface of a hemisphere of radius 6 cm and refractive index 1.5, placed in air as shown in figure-(i). Assume paraxial ray approximation:

(A*) The rays are focussed at 12 cm from the point \(P\) to the right, in the situation as shown in figure-(i)

(B) The rays are focussed at 16 cm from the point \(P\) to the right, in the situation as shown in figure-(i)

(C) If the rays are incident at the curved surface (figure-(ii)) then these are focused at distance 18 cm from point \(P\) to the right.

(D*) If the rays are incident at the curved surface (figure-(ii)) then these are focused at distance 14 cm from point \(P\) to the right.

Sol.

Figure (i) 

\[\mu=\frac{3}{2}\]

Surface (1):
\[
\frac{3}{2} - \frac{1}{\infty} = \frac{3}{2} - 1
\]
\[\therefore \quad v_1 \rightarrow \infty\]
Surface (2):
\[
\frac{3}{2} - \frac{3/2}{\infty} = \frac{1-3/2}{(-6)} \Rightarrow r_2 = +12\]

Figure (ii)

\[\mu=\frac{3}{2}\]

Surface (1):
\[
\frac{3}{2} - \frac{1}{\infty} = \frac{3/2 - 1}{(+6)}
\]
\[v_1 = +18\text{ cm}\]
Surface (2):
\[
\frac{1}{v_2} - \frac{3/2}{(+12)} = \frac{1-3/2}{\infty}
\]
\[v_2 = +8\text{ cm}\]

option (A) and option (B) are correct.

7. A point source of light is placed at a distance \(h\) below the surface of a large deep lake.

(a) Show that the fraction \(f\) of the light energy that escapes directly from the water surface is independent of \(h\) and is given by:
\[
f = \frac{1}{2} \left( 1 - \frac{1}{2n} \sqrt{n^2 - 1} \right)
\]
where \(n\) is the index of refraction of water.

(Note: Absorption within the water and reflection at the surface; except where it is total, have been neglected)

(b) Evaluate this ratio for \(n = 4/3\). \(\text{Ans.} \left(4 - \frac{\sqrt{7}}{2}\right)/8\)

Sol.

\[n + \sin \theta = 1 \times \sin 90^\circ\]
\[\sin \theta = \frac{1}{n}\]
\[ \cos \theta = \sqrt{1 - \frac{1}{n^2}} \]

Note:
Fore sphere

\[ dA = (2\pi R \cos \phi) (R d\phi) \]
\[ dA = 2\pi R^2 \cos \phi d\phi \]

For area of out side sphere, \( \phi \) varies from \((90 - \theta)\) to \(90^\circ\)

\[ \therefore \text{ Aarea of side water} = \int_{90 - \theta}^{90} 2\pi R^2 \cos \phi d\phi \]

\[ = 2\pi R^2 \int_{90 - \theta}^{90} \cos \phi d\phi \]
\[ = 2\pi R^2 [\sin 90 - \sin (90 - \theta)] \]
\[ = 2\pi R^2 [1 - \cos \theta] \]

Area of side water = \(2\pi R^2 [1 - \sqrt{1 - \frac{1}{n^2}}] \)

Practition:

\[ F = \frac{\text{Area of sphere outside water}}{\text{Total area of sphere}} \]

\[ F = \frac{2\pi R^2}{4\pi R^2} \left[ 1 - \sqrt{1 - \frac{1}{n^2}} \right] \]

\[ F = \frac{1}{2} \left[ 1 - \sqrt{1 - \frac{1}{n^2}} \right] = \frac{1}{2} \left( 1 - \frac{1}{2n} \sqrt{(n^2 - 1)} \right) \]

(b) \[ n = 4/3 \Rightarrow F = \frac{4 - \sqrt{7}}{8} \]

8. A ray of light is incident from air to a glass rod at point A. The angle of incidence being \(45^\circ\). The minimum value of refractive index of the material of the rod so that T.I.R. takes place at B is_____.

Ans. \([\sqrt{3}/2]\)

Sol.

\[ r = 90 - \theta \Rightarrow \theta = 90 - r \geq \frac{\theta}{2} \]
\[ \sin (90 - r) \geq \sin \theta \]
\[ \cos r \geq \frac{1}{n} \]
\[ 1 - \sin^2 r \geq \frac{1}{n} \Rightarrow 1 - \frac{1}{2n^2} \geq \frac{1}{n^2} \]
\[ n \geq \sqrt{(3/2)}, \quad n_{\min} = \sqrt{(3/2)} \quad \text{Ans.} \]

9. A man observes a coin placed at the bottom of a beaker which contains two immiscible liquids of refractive indices 1.2 and 1.4 as shown in the figure. A plane mirror is also placed on the surface of liquid. The distance of image (from mirror) of coin in mirror as seen from medium A of refractive index 1.2 by an observer just above the boundary of the two media is:

(A) 18 cm \quad (B^*) 12 cm \quad (C) 9 cm \quad (D) None of these
Image of coin due to $AB$ interface is at depth $\left( \frac{4}{1.4} \right) \times 1.2$ from interface $AB$.

For light moving from nod $A$ to air image is at $\left[ \frac{7}{1.4} \times 1.2 + 3 \right] \times \frac{1}{1.2}$ from mirror and nod $A$ interface.

Now final image will from observer will be at

$$\left\{ \left[ \frac{7}{1.4} \times 1.2 + 3 \right] \times \frac{1}{1.2} \right\} \times 1.2 + 3 = \left[ 6 + 3 \right] + 3 = 12 \text{ cm}$$

10. A container contains water up to a height of 20 cm and there is a point source at the centre of the bottom of the container. A rubber ring of radius $r$ floats centrally on the water. The ceiling of the room is 2.0 m above the water surface. (a) Find the radius of the shadow of the ring formed on the ceiling if $r = 15$ cm. (b) Find the maximum value of $r$ for which the shadow of the ring is formed the ceiling. Refractive index of water $= 4/3$.

**Ans.** (a) $\frac{169}{60} m = 2.8 \text{ m}$; (b) $\frac{3}{5\sqrt{7}} m = 22.6 \text{ cm}$

**Sol.**

$$\mu \sin \theta = \frac{1}{\sin r}$$

$$\sin r = \frac{3}{5}$$

$$r' = 53^\circ$$

$$\tan 53^\circ = \frac{x}{2}$$

$$\frac{4}{3} = \frac{x}{2} \Rightarrow x = \frac{8}{3} \text{ m}$$

Net radius will be

$$\frac{15}{100} + \frac{8}{3} = 2.8 \text{ m}$$

Shadow will form till critical incidence

$$\theta_c = \sin^{-1} \left( \frac{3}{4} \right)$$

i.e.,

$$\tan \theta = \frac{r}{20}$$

$$\therefore \quad \sin \theta = \frac{3}{4}$$

$$\Rightarrow \quad \tan \theta = \frac{3}{\sqrt{7}}$$

$$\therefore \quad r = \frac{3}{\sqrt{7}} \times 20 \text{ cm or } \frac{3}{5\sqrt{7}} \text{ m}$$
1. A small rod $ABC$ is put in water making an angle $6^\circ$ with vertical. If it is viewed paraxially from above, it will look like bent shaped $ABC'$. The angle of bending ($\angle CBC'$) will be in degree ... \( n_w = \frac{4}{3} \):

(A*) $2^\circ$

(B) $3^\circ$

(C) $4^\circ$

(D) $4.5^\circ$

Sol.

\[ \alpha = \frac{v}{OC}, \beta = \frac{v}{OC'} \]

\[ \frac{\alpha}{\beta} = \frac{OC}{OC'} = \frac{1}{\mu} \]

\[ \beta = \mu \alpha = \left( \frac{4}{3} \right) (6^\circ) = 8^\circ \]

\[ \beta - \alpha = 2^\circ \]

2. If the observer sees the bottom of vessel at $8$ cm, find the refractive index of the medium in which observer is present.

Ans. \( \left[ \frac{16}{15} \right] \)

Sol.

\[ \mu_{\text{sel}} = \frac{10}{8} \]

\[ \frac{4/3}{\mu} = \frac{10}{8} \Rightarrow \mu = \frac{4}{3} \times \frac{8}{10} = \frac{16}{15} \]

3. A man starting from point $P$ crosses a $4$ km wide lagoon and reaches point $Q$ in the shortest possible time by the path shown. If the person swims at a speed of $3$ km/hr and walks at a speed of $4$ km/hr, then his time of journey (in minutes) is ?

Ans. [250]

Sol.

\[ \sin \theta_1 = \frac{3}{5} \]

\[ \frac{\sin \theta_2}{v_2} = \frac{\sin \theta_1}{v_1} \]

\[ \Rightarrow \sin \theta_2 = 4 \times \frac{315}{3} = 400 \]
\[ \sin \theta_2 = \frac{4}{5} \]

distance travelled in water \[ = 5 \text{ km} \]

time taken in water \[ = \frac{5}{3} \times 60 \text{ hr} \]
\[ = 100 \text{ min} \]

distance travelled in ground \[ = \frac{6}{\theta_2} = \frac{6}{3/5} \]

time \[ = \frac{10 \times 60}{\theta} = 150 \text{ min.} \]
\[ \Rightarrow 150 + 100 = 250 \text{ min.} \]

4. The figure shows a parallel slab of refractive index \( n_2 \) which is surrounded by media of refractive indices \( n_1 \) and \( n_3 \). Light is incident on the slab at angle of incidence \( \theta \neq 0 \). The time taken by the ray to cross the slab is \( t_1 \) if incidence is from \( n_1 \) and it is \( t_2 \) if the incidence is from \( n_3 \). Then assuming that \( n_2 > n_1, n_2 > n_3 \) and \( n_3 > n_1 \), then value of \( t_1/t_2 \):

(A) = 1  
(C*) < 1 
(D) Cannot be decided  
(B) > 1

\[ n_1 \sin \theta = n_2 \sin \theta_1 \]
\[ \Rightarrow \sin \theta_1 = \frac{n_1}{n_2} \sin \theta \]
\[ n_3 \sin \theta = n_2 \sin \theta_2 \]
\[ \Rightarrow \sin \theta_2 = \frac{n_3}{n_2} \sin \theta \]

But \( n_3 > n_1 \)
\[ \Rightarrow \sin \theta_2 > \sin \theta_1 \]
thus \( t_2 > t_1 \)
and \( \frac{t_1}{t_2} < 1 \)

5. A pole of length 2.00 m stands half dipped in a swimming pool with water level 1 m higher than the bed (bottom). The refractive index of water is \( 4/3 \) and sunlight is coming at an angle of 37º with the vertical. Find the length of the shadow of the pole on the bed: Use \( \sin^{-1} (0.45) = 26.8^\circ, \tan (26.8^\circ) = 0.5 \).

Ans. [1.25 m]

\[ \sin 37^\circ = \frac{4}{3} \sin r \]
\[ \frac{3}{5} = \frac{4}{3} \sin r \]
\[ \Rightarrow \sin r = \frac{9}{20} \]
\[ \Rightarrow r = 26.8^\circ \]
Length of shadow \[ = \frac{3}{4} + 1 \times \tan 26.8^\circ \]
\[ = 0.75 + 0.5 \]
\[ = 1.25 \text{ m} \]
6. A ray of light is incident from air to a glass rod at point $A$. The angle of incidence being $45^\circ$. The minimum value of refractive index of the material of the rod so that T.I.R. takes place at $B$ is ______:

(A) $\sqrt{1.2}$
(B) $\sqrt{1.5}$
(C) $\sqrt{1.7}$
(D) $\sqrt{2.3}$

**Sol.**

For surface $B$

$\mu \sin (90 - \theta) = 1 \sin 90^\circ$

$\mu \cos \theta = 1$

$\cos \theta = \frac{1}{\mu}$  ...(1)

For surface $A$

$\frac{1}{\sqrt{2}} \mu \sin \theta$

$\sin \theta = \frac{1}{\sqrt{2} \mu}$  ...(2)

From (1) and (2)

$\tan \theta = \frac{\frac{1}{\sqrt{2} \mu}}{\frac{1}{\mu}} = \frac{\sqrt{2}}{1}$

$\tan \theta = \frac{1}{\sqrt{2}}$

$\mu = \frac{1}{\cos \theta} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{2} = \sqrt{1.5}$  Ans.

7. In the figure shown a slab of refractive index $\frac{3}{2}$ is moved toward a stationary observer. A point 'O' is observed by the observer with the help of paraxial rays through the slab. Both 'O' and observer lie in air. The velocity with which the image will move is:

(A) 2 m/s towards left
(B) $\frac{4}{3}$ m/s towards left
(C) 3 m/s towards left
(D) Zero

**Sol.**

$\mu = \frac{3}{2}$

Shift in the position of image is independent of the position of slab hence due to movement of slab object do not gain any velocity.

8. A beam of light is incident on a spherical drop of water at an angle $i$. Find the angle between the incident ray & the emergent ray after one reflection from internal surface. Is this possible by total internal reflection?

**Ans.** $[\delta = 2i - 4 \sin^{-1} \left( \frac{3}{4} \sin i \right) + \pi, \ No]$

**Sol.**

For first deviation $S_1 = i - r$

For second deviation $S_2 = \pi - 2r$

For third deviation $S_3 = i - r$

Total deviation will be
\[ \delta = S_1 + S_2 + S_3 = 2i + \pi - 4r \] ...(1)

and from Snell's law

\[ \sin i = \mu \sin r \]

\[ \sin i = \frac{4}{3} \sin r \]

\[ \alpha = \sin^{-1} \left( \frac{3}{4} \sin i \right) \] ...(2)

Hence from (1) and (2)

\[ \delta = 2i + \pi - 4 \sin^{-1} \left( \frac{3}{4} \sin i \right) \]

9. A glass hemisphere of refractive index 4/3 and of radius 4 cm is placed on a plane mirror. A point object is placed on axis of this sphere at a distance \( d' \) as shown. If the final image is formed at infinity, then find the value of \( d' \) in cm.

**Ans.** [3 cm]

**Sol.**

Image of object \( O \) due to refraction at \( AB \) is at \( du \) from surface \( AB \)

Now due to \( ACB \) surface.

\[ \frac{1}{v} - \frac{\frac{4}{3}}{(4 + du)} = \frac{1 - 4/3}{-4} \]

\[ v = \frac{d - 9}{12(3 + d)} \]

Now suppose \( d > 9 \) then image is formed below the mirror at distance \( v' = \frac{d - 9}{(12)(3 + d)} \) from mirror

Now this image of mirror will act as object for refraction again at surface \( ACB \) (light goes from air to water)

So,

\[ \frac{4}{3v} - \frac{(d - 9)}{(12)(3 + d)} = \frac{4/3 - 1}{4} = \frac{1}{12} \]

For final image to be at infinity \( v = \infty \)

So,

\[ \frac{-(d - 9)}{12(3 + d)} = \frac{1}{12} \Rightarrow -d + 9 = 3 + d \]

\[ = 2d - C \Rightarrow d = 3 \text{ cm} \]

10. An observer observes a fish moving upwards in a cylindrical container of cross section area 1 m² filled with water up to a height of 5 m. A hole is present at the bottom of the container having cross section area 1/1000 m². Find out the speed of the fish observed by observer when the bottom hole is just opened. (Given: The fish is moving with the speed of 6 m/s towards the observer, \( \mu \) of water = 4/3) **Ans.** [4.4975 m/s]

**Sol.**

Velocity of liquid from the hole is 10 m/sec.

So using equation of continuity

\[ A_1 V_1 = A_2 V_2 \]

\[ d = 5 \text{ m} \]

\[ v = \sqrt{2gh} \]
\[ U_2 = \frac{1}{10} \text{ m/sec} = 1 \text{ cm/sec} \]

\[ y = \frac{x}{\mu} \]

\[ \frac{dy}{dt} = \frac{1}{\mu} \left( \frac{dx}{dt} \right) \]

Here \( x \) decreases so \( \frac{dx}{dt} \) is \(-601 \text{ cm/sec}\)

\[ \frac{3}{4} \times (-601) = -450.75 \text{ m/sec} \]

This \( \frac{dy}{dt} \) is velocity of image from surface so velocity of image will be \( 450.75 - 1 \)

\[ = 449.75 \text{ cm/sec} = 4.4975 \text{ m/sec} \]